MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023 (TERM ID: 2230)

PCM0235 – CALCULUS

(Foundation in Information Technology)

2 OCTOBER 2023
9:00 a.m. –11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of 4 **pages** (including cover page) with 5 Questions only.

2. Attempt all the questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1 [10 marks]

(a) Function $g(x)$ is a function that is continuous everywhere and defined as follows:

$$g(x)=\begin{cases}3x^2-1, & x>3\\ a(2x-3), & x\le 3\end{cases}$$

Find the value of a. [3 marks]

(b) Evaluate the following limit:

(i) $\lim\limits_{x\to 2}\left(\dfrac{2x+1}{x^2+3}\right)$ [2 marks]

(ii) $\lim\limits_{x\to\infty}\dfrac{x^2+1}{x^3-3x+2}$ [3 marks]

(iii) $\lim\limits_{x\to 4}\dfrac{2x-8}{x^2-16}$ [2 marks]

Question 2 [10 marks]

(a) Find $\int\dfrac{3x}{x^2+1}\,dx$ [3 marks]

(b) Find $\int 3xe^{4x}dx$ [3 marks]

(c) Find $\int\dfrac{5}{(x-2)(x-1)}\,dx$ [4 marks]

Question 3 [10 marks]

(a) Find the derivative of the following function:

$$f(x)=\frac{x^2+e^x}{x^3+2x}$$ [3 marks]

Continued...

(b) Find the first, second and third derivatives of the following function.

$$y = 4x^4 - 12x^2 + 4e^{2x} + 5$$ [3 marks]

(c) Use implicit differentiation to find $\frac{dy}{dx}$ if $x^3 + 4xy = 6y$. [4 marks]

Question 4 [10 marks]

(a) Let $f(x) = 4x^4 - 3x^2 + 2$.

(i) Find $f'(x)$. [1 mark]

(ii) Identify the critical point of $f(x)$ [2 marks]

(iii) Determine the function's relative extreme values, if any, saying where they occur. [3 marks]

(b) A spherical balloon is increasing at the rate of 50 m^3/min. Given that volume of a sphere is $V = (4/3)\pi r^3$, surface area of sphere, $A = 4\pi r^2$.

(i) How fast is the balloon's radius increasing at the instant the radius is 3 meter? [2 marks]

(ii) How fast is the surface area increasing? [2 marks]

Question 5 [10 marks]

(a) Find the solution of second-order initial value problem.

$$y'' + 6y' + 9 = 0, \qquad y(0) = 1, y'(0) = -2$$ [4 marks]

(b) Sketch the graph enclosed by given curves $x = y^2 + 1$ and $x = y + 1$. Find the volume of the solids generated by revolving the regions bounded by $x = y^2 + 1$ and $x = y + 1$ about the y-axis. [6 marks]

Continued...

APPENDIX

The derivative of a function f at a number x, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

The Product Rule, $\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$

The Quotient Rule, $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Derivatives of Trigonometric, Logarithmic and Exponential Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$ $\quad$ $\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[a^x] = a^x \ln a$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

The Chain Rule

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ or $\frac{d}{dx}[f(u)] = f'(u).\frac{du}{dx}$

Integration of Trigonometric functions

$\int \sin x \, dx = -\cos x + C$ $\quad$ $\int \cos x \, dx = \sin x + C$ $\quad$ $\int \tan x \, dx = \ln|\sec x| + C.$

$\int \sec^2 x \, dx = \tan x + C$ $\quad$ $\int \sec x \tan x \, dx = \sec x + C$ $\quad$ $\int \csc^2 x \, dx = -\cot x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

End of paper.